EP2.3

Universality on Network Structures

- From quantum theory to big data -

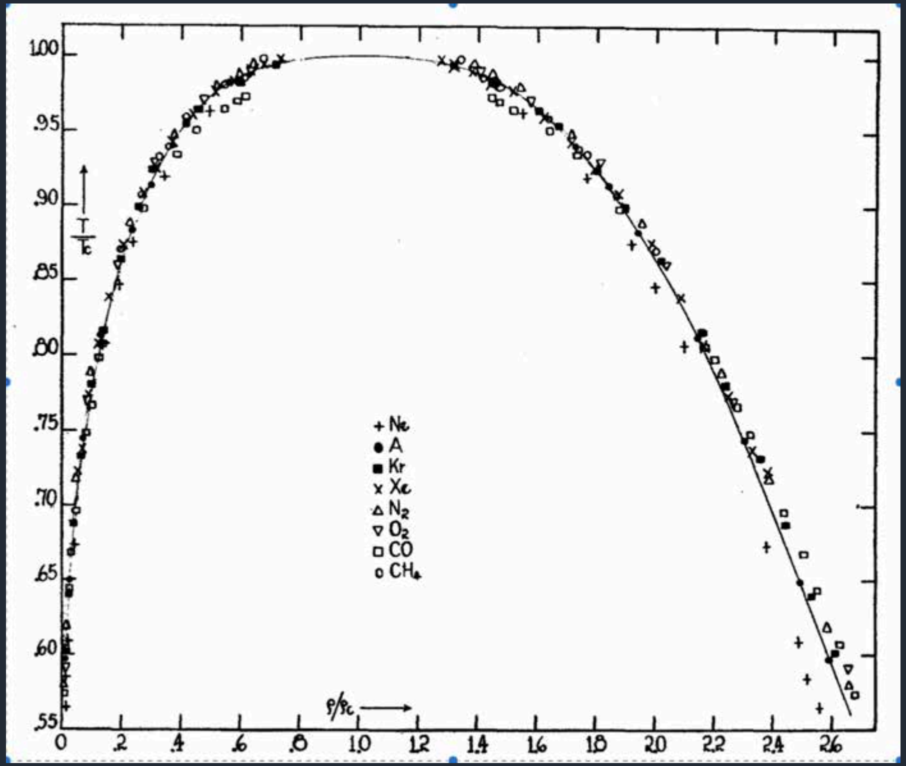

STRUCTURES CLUSTER OF EXCELLENCE
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386
STRUCTURES
A UNIFYING APPROACH TO EMERGENT PHENOMENA IN THE PHYSICAL WORLD, MATHEMATICS, AND COMPLEX DATA

Structures

Universality

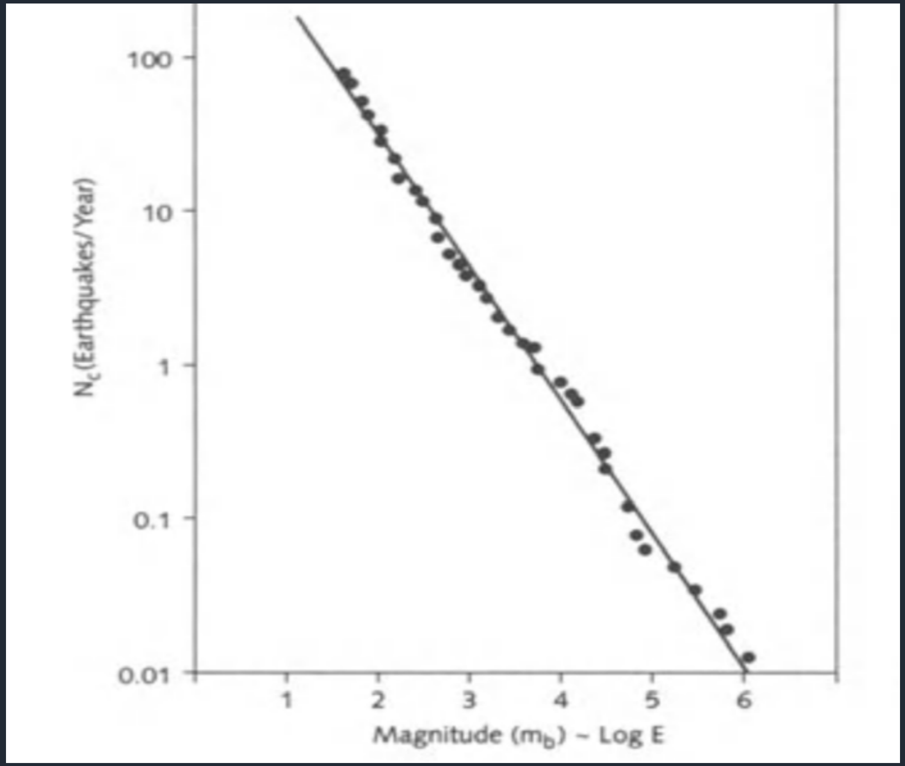
In statistical physics

Emergence of power law behaviour for thermodynamic quantities close to a critical point.



In dynamical systems

Observation of ubiquitous patterns in nature: fractals, 1/f noise, scaling of catastrophic events.



1) E. A. Guggenheim J. Chem. Phys. 13, 253 (1985)
2) P. Bak, How Nature Works, Springer (1996)

“Rigorous” Definition

$$\Psi \propto \varepsilon^{\beta}$$

$$O(N)$$

$$\mathcal{D} \propto \varepsilon^{\frac{d-2}{2}}$$

Order parameter

The order parameter captures the continuum behaviour and reflects the symmetry changes in the system as a parameter is varied

Symmetry

Universal behaviour only depends on the symmetry of the order parameter

Low-Energy DOS

The universal properties only depend on the density of states for low-energy excitations

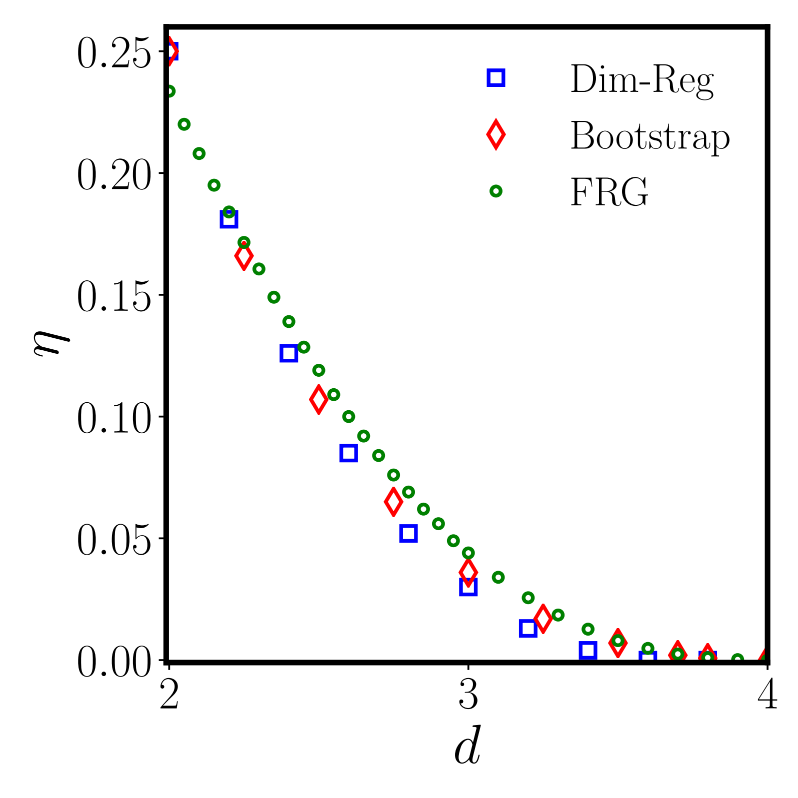
A continuous model exists!

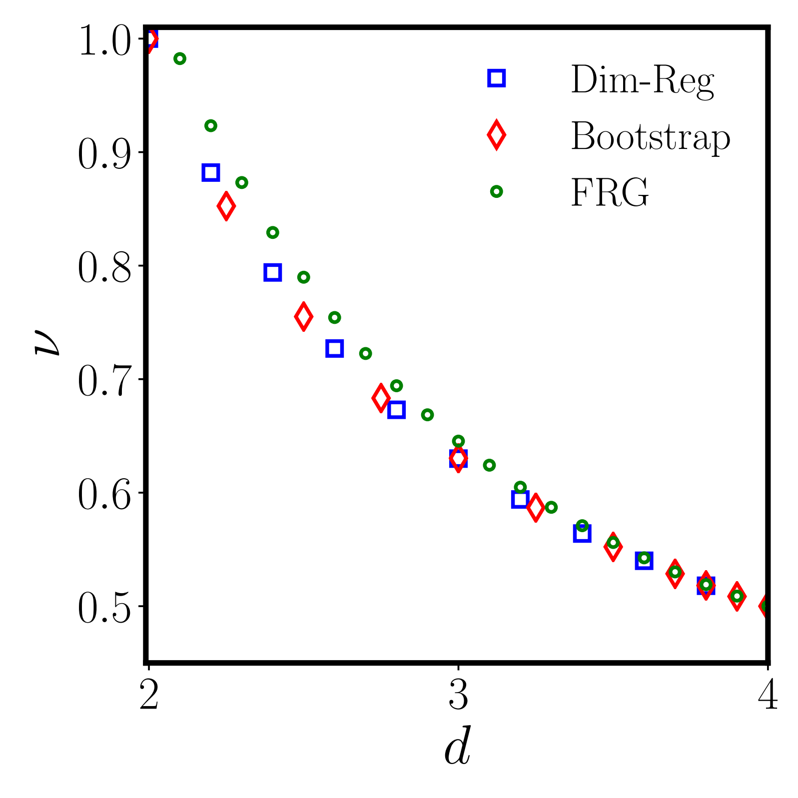
Field Theory

$$H = \int d^d x \left\{ \partial_\mu \psi \partial_\mu \psi + m\psi^2 + g\psi^4 \right\}$$

$$\xi \propto \varepsilon^{-\nu}$$

$$\langle \psi(0)\psi(r) \rangle \propto r^{d-2+\eta}$$





1) Y. Holovatch, Theor. Math. Phys. 96, 1099–1109 (1993).
2) S. El-Showk et al., J. Stat. Phys. 157, 869–914 (2014).
3) A. Codello et al., Phys. Rev. D 91, 105003 (2015).

What is the meaning of non-integer dimensions?

The Spectral Dimension

Vibrational spectrum

At low frequencies the vibrational spectrum of coupled oscillators on a network obeys:

$$\rho(\omega) \propto \omega^{d_s - 1}$$

Random Walk Return Rates

A random walker hopping on networks will return at its starting point with probability:

$$P_0(t) \propto t^{-d_s/2}$$

Mermin-Wagner Theorem

Spontaneous symmetry breaking of continuous symmetries is forbidden for

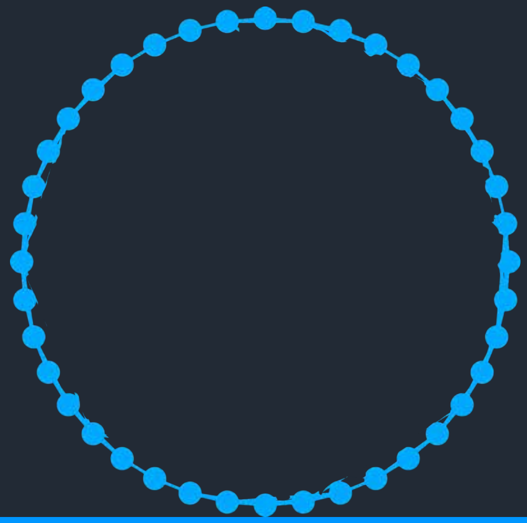
$$d_s \leq 2$$

Does it control universal properties?

What about discrete symmetries?

1) R. Burioni, D. Cassi, Phys. Rev. Lett. 76, 1091 (1996).
2) R. Burioni, D. Cassi, A. Vezzani, Phys. Rev. E 60, 1500 (1999).

Prototypical network



Additional bonds

Turn on bonds between sites with distance dependent probability

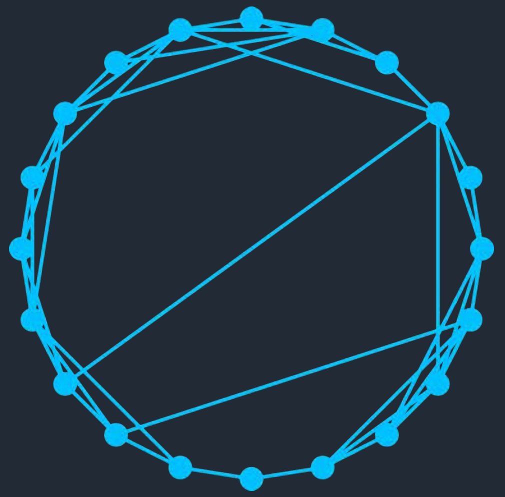
$$P_{ij} \propto \frac{1}{|i-j|^{1+\sigma}}$$

$$\bar{a}_{ij} = 1/r_{ij}^{1+\sigma}$$

Ring Structure

One dimensional lattice with periodic boundary condition:

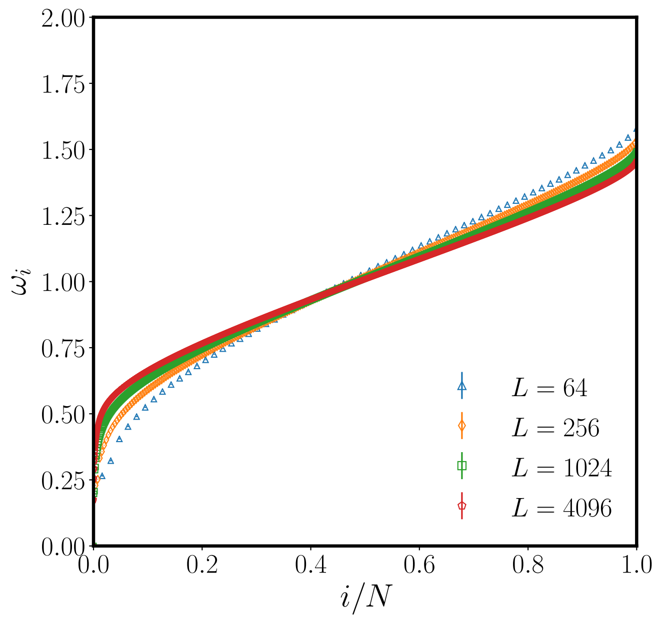
$$d_s = 1$$



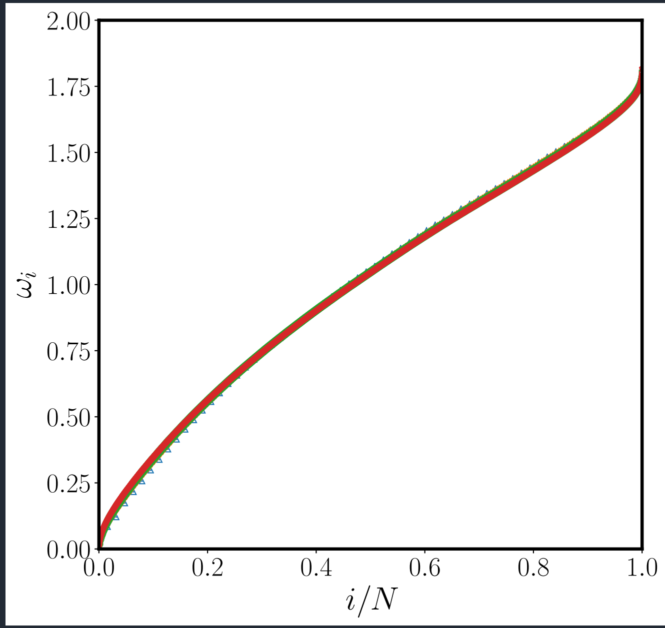
“Mean-Field” model

Consider the spectrum of the average graph, rather than the average of the spectra

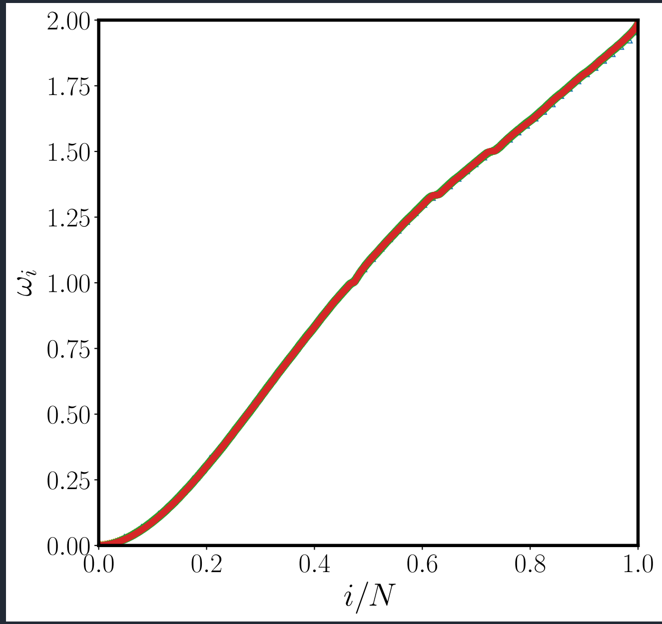
Spectra



$\sigma = 0.0$



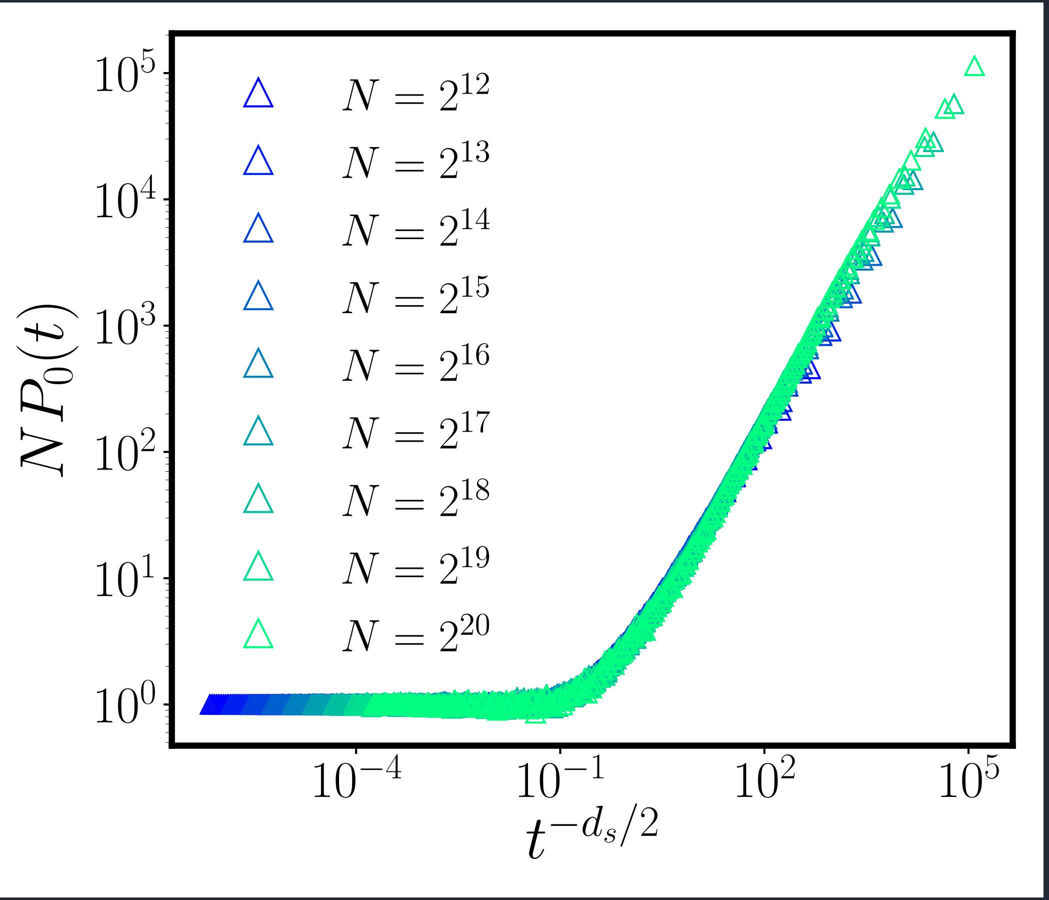
$\sigma = 0.5$



$\sigma = 1.5$

Return rate collapse

$$P_0^N(t) = \frac{1}{N} f\left(N t^{-d_S/2}\right)$$



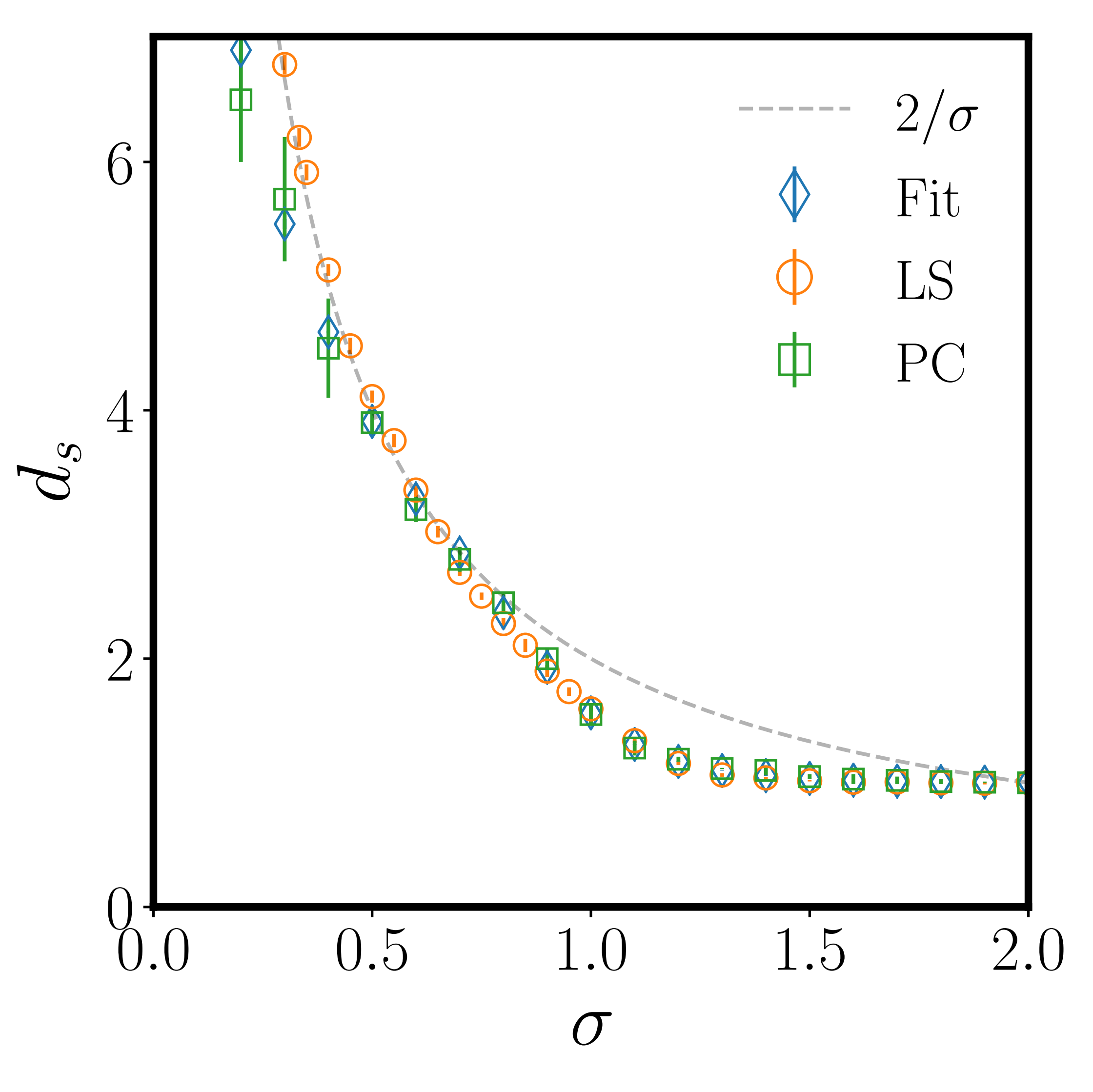
Tuneable spectral dimension

Collapse of the return probabilities

The average of the random walker return rates for several network realisations with different number of sites are collapsed to obtain an estimation of the spectral dimension.

Scaling of the smallest eigenvalues

The power law scaling of the first eigenvalue of the laplacian operator as a function of the network size is studied for large network sizes.



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Scaling Theory

$$H_{\mathrm{lr}} = \int d^d x \left\{ \partial_\mu^{\frac{\sigma}{2}} \psi \partial_\mu^{\frac{\sigma}{2}} \psi + \mu \psi^2 + g \psi^4 \right\}$$

vs

$$H_{\mathrm{sr}} = \int d^{d_{\mathrm{eff}}} x \left\{ \partial_\mu \psi \partial_\mu \psi + \mu \psi^2 + g \psi^4 \right\}$$

At one-loop, the“same” universal properties occur if

$$d_{\mathrm{eff}} = (2 - \eta_{\mathrm{sr}}) \frac{d}{\sigma}$$

1) G. S. Joyce Phys. Rev. 146, 349 (1966).
2) M. C. Angelini, et al. Phys. Rev. E 89, 062120 (2014).
3) N. Defenu, et al. Phys. Rev. E 92 (5), 052113 (2015).

Anomalous Dimension of LR disorder?

"Mean-Field" Formula

$$d_s = \begin{cases} 2/\sigma & \sigma < 2 \\ 1 & \sigma \geq 2, \end{cases}$$

Definition of η

$$d_s = \frac{2-\eta}{\sigma}$$

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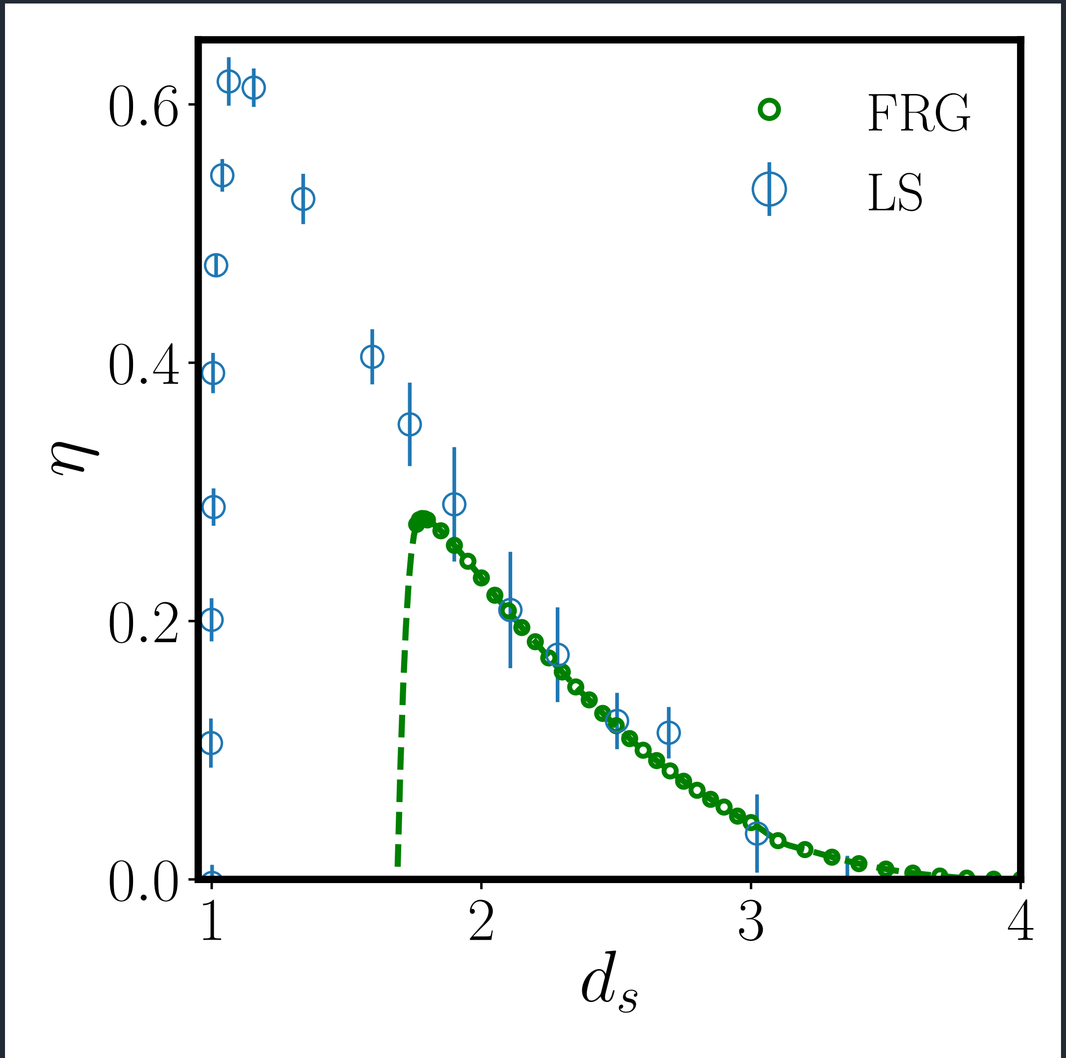


Why classical systems?

Big Data

Principal components analysis (PCA) focuses on the eigenvalues of the covariance matrix to describe large data-sets. Strong correlation beyond PCA may emerge in the big-data limit depending on the spectral dimension.

Network Dynamics

The study of diffusion, synchronisation and epidemic spreading on complex networks displays several universal features and it is strongly effected by the existence of a finite spectral dimension.

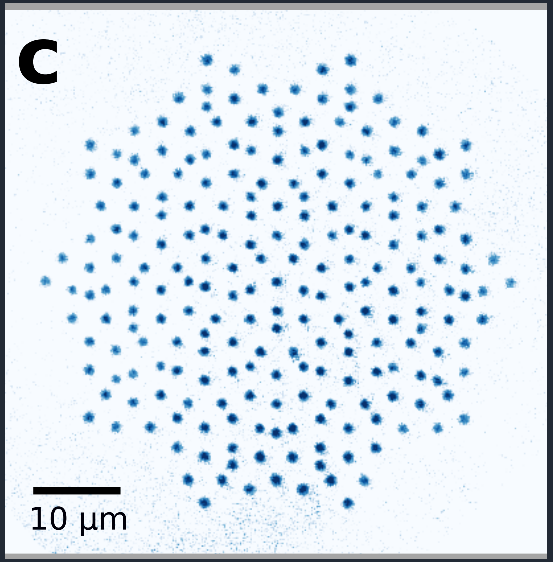
- Brain Modelling
- Social Interaction Networks
- Financial Networks
- Engineered Materials
- Quantum Gravity

1) S. Bradde, W. Bialek, J. Stat. Phys. 167, 462 (2017).
2) I. Iacopini, et al., Nat. Comm. 10, 2485 (2019).
3) M. Tumminello, et al., Proc. Nat. Acad. Sci. 102, 10421 (2005).
4) D. Bassett, et al., Phys. Rev. E 86, 041306 (2012).
5) A. P. Millán, et al., Sci. Rep. 8, 9910 (2018).

Why quantum systems?

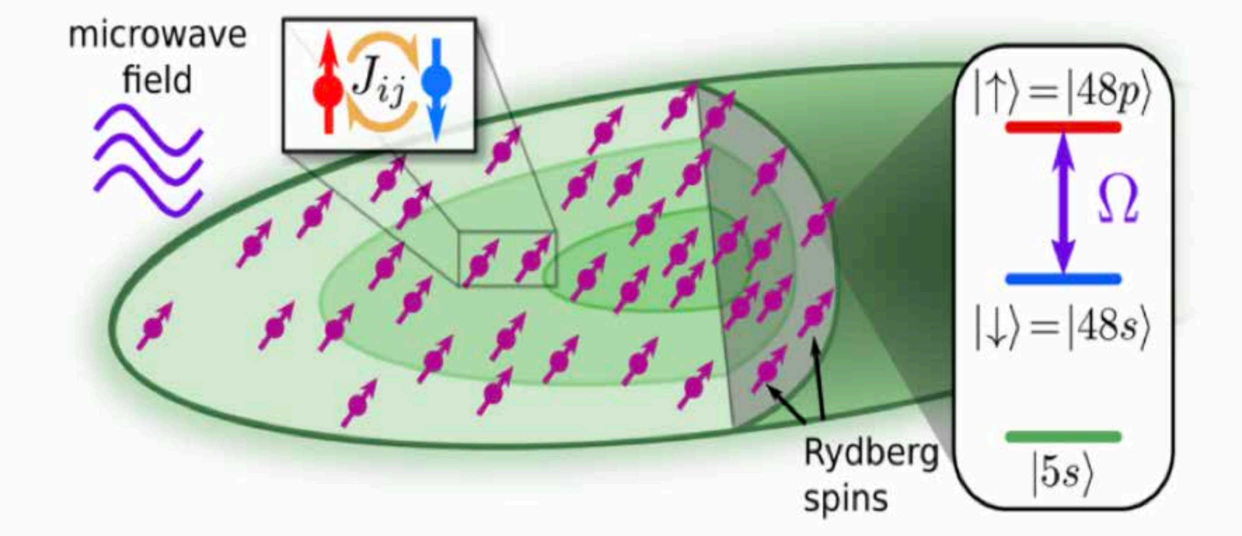
Quantum engineering

Quantum technologies require accurate quantum state preparation. However, quantum fluctuations limit the possible accuracy of experimental manipulations. These limitations are often universal but may be reduced by engineering systems with real spectral dimensions.



Rydberg Gases

Rydberg gases are generated by shining an off resonant laser field on a cold atom ensemble. Due to the Rydberg blockade effect, these systems display a highly non-homogeneous structure.



1) Y. Wang, et al., arXiv:1912.04200 (2019).
2) A. Orioli et al., Phys, Rev. Lett. 120, 63601 (2018).

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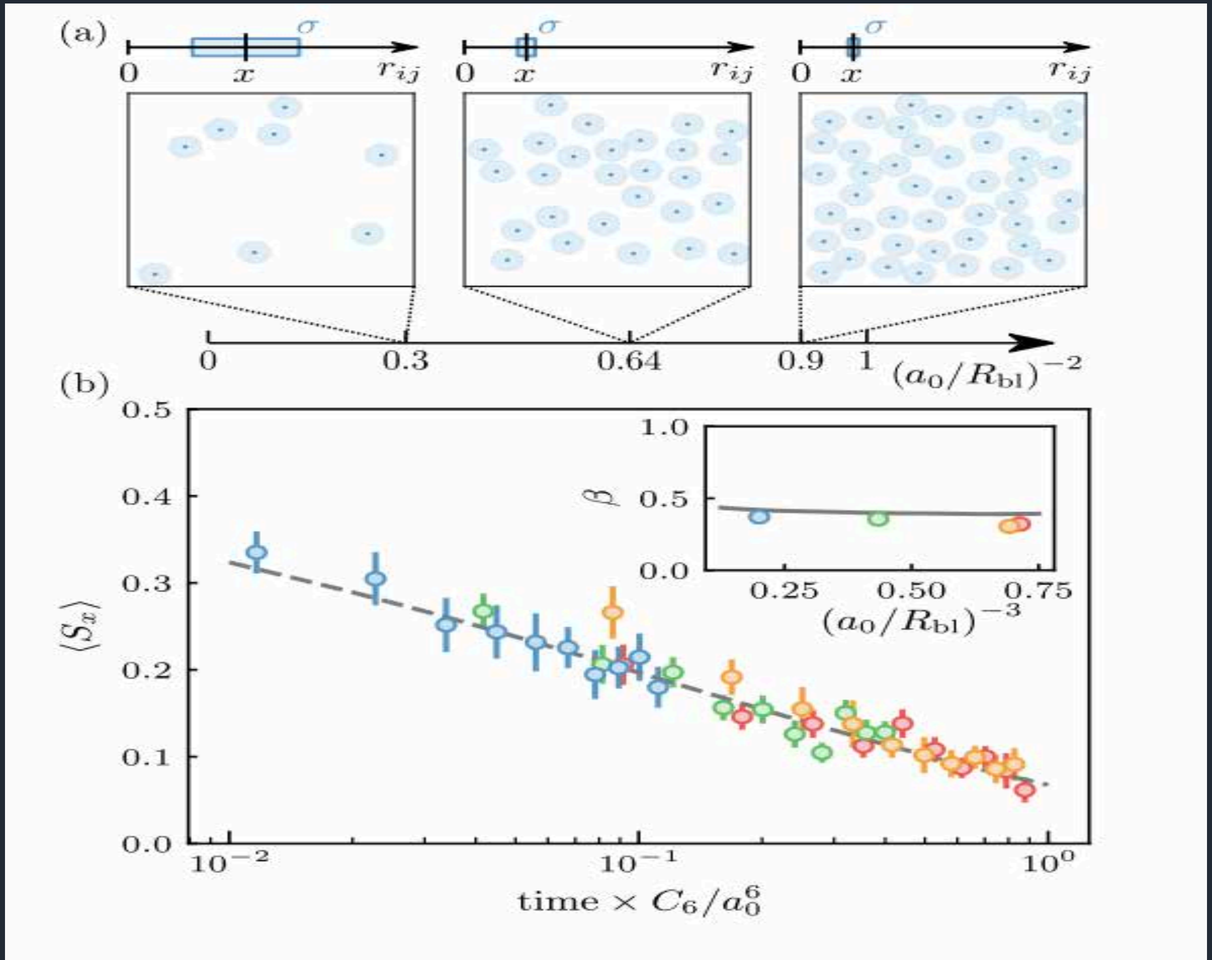
Why dynamical universality?

Self organised criticality in cold atoms

In the dissipative regime the dynamics of a Rydberg atom ensemble can be related to the one of classical systems. Dynamical universality is then related to self-organised criticality.

Statistical physics goes quantum

Traditional dynamical problems of statistical physics, such as random walks or percolation, can be now investigated within the unitary dynamics paradigm.



1) M. Buchold, et al., Phys. Rev. B 95, 014308 (2017).
2) A. Signoles, T. Franz et al., arXiv:1909.11959.
3) S. Helmrich, et al., arXiv:1806.09931 (2018).

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Thank you!

arXiv:2006.10421