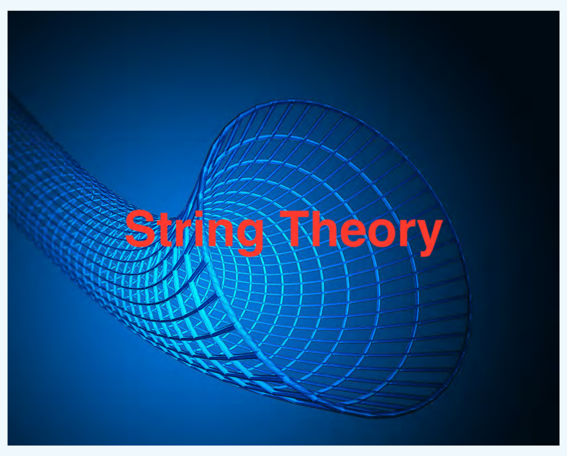


Geometry and Physics of BPS states in String Theory and QFT

Pietro Longhi
ETH Zürich

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research
FNSNF
Swiss National Science Foundation

Defects as BPS probes

Defects such as domain walls or Wilson lines provide interesting ways of probing the dynamics of quantum field theories, sometimes providing information that would otherwise be invisible to local operators.

We focus on the study of codimension-two half-BPS defects in supersymmetric gauge theories with eight supercharges in five dimensions. BPS states organize into a 3d sector, a 5d sector, and a mixed 3d-5d sector.

Goals

String theory provides a top-down view of coupled 3d-5d systems, allowing to rephrase the study of their BPS spectra in terms of a certain class of problems in enumerative geometry. Our work aims to exploit this feature to achieve three main goals

1. Develop new computational frameworks for 3d, 5d and 3d-5d BPS states.
2. Uncover novel structures in BPS spectra, such as quiver descriptions.
3. Explore relations and interactions among different sectors.

Background

We consider 3d-5d systems that arise as effective descriptions of M theory on a spacetime $X \times S^1 \times \mathbb{R}^4$ with X a toric Calabi-Yau threefold, in presence of one or more M5 branes wrapping a Lagrangian submanifold L.

BPS states of the 5d theory $T[X]$ consist of instanton-dyons arising from M2 branes wrapped on compact 2-cycles of X, as well as monopole strings arising from M5 branes wrapping compact 4-cycles. Compactification on S^1 reduces this setup to type IIA String Theory, whose BPS spectrum is captured by generalized Donaldson-Thomas (DT) invariants of X.

Computing DT invariants for a given choice of X is generally a challenging problem in mathematics. In String Theory we approach this problem through the connection to $T[X]$. The 5d theory is coupled to a defect theory $T[L]$ engineered by an M5 brane wrapping L. The low-energy dynamics of the resulting 3d-5d system leads naturally to the notion of *Exponential Networks* [8, 7] for the mirror curve of the system (X, L), identified with the (twisted) chiral ring of $T[L]$ on $S^1 \times \mathbb{R}^2$, as well as the Seiberg-Witten curve of $T[X]$.

When $X = \mathcal{O}(-1)^{\oplus 2} \to \mathbb{P}^1$, one may choose $L = L_K$ as the conormal Lagrangian of an arbitrary knot K. Open topological strings compute symmetrically colored HOMFLY-PT polynomials of K, establishing a connection between BPS states of $T[L_K]$ and knot theory [5]. It was recently observed that this sector of the BPS spectrum has an interesting structure in terms of *quivers* [6].

3d, 5d and 3d-5d

BPS states organize into a 5d sector related to $T[X]$, a 3d sector related to $T[L]$, and a more exotic mixed 3d-5d sector.

Exponential networks are primarily concerned with the study of the novel 3d-5d BPS spectrum. Their geometric definition is motivated naturally by String Theory, [8, 7]. In [3, 2] we develop a new approach to compute 3d-5d BPS states as well as 5d BPS states based on exponential networks. A key idea is to adopt a field-theoretic viewpoint to take advantage of the connection to 3d tt^* geometry, inspired by related phenomena in 2d-4d systems [9].

3d BPS states of $T[L]$ are captured by open A-model topological string wavefunctions. The knot-quiver correspondence conjectures that, if (X, L_K) is a knot conormal, the 3d BPS spectrum can be packaged into the finite data of a quiver. In [4, 1] we elucidated the physical and mathematical origins of this correspondence. This led us to conjecture a much broader range of validity, as well as to the discovery of a new class of 3d $\mathcal{N} = 2$ dualities.

Exponential Networks

Example: $X = \mathcal{O}(-1)^{\oplus 2} \to \mathbb{P}^1$

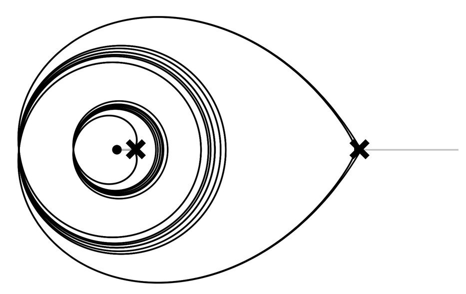
$$\Sigma = \{(x, y) \in \mathbb{C}^* \times \mathbb{C}^* \,|\, 1 + y + xy + Qxy^2 = 0\}$$

with BPS trajectories

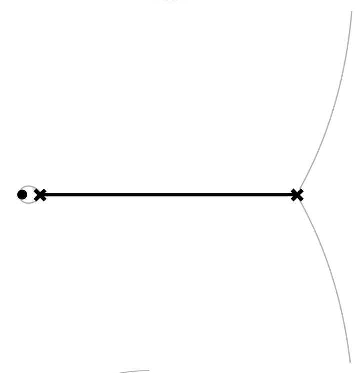
$$(\log y_+(x) - \log y_-(x) + 2\pi i N)\,\frac{d\log x}{dt} = e^{i\vartheta}$$

Topological jumps at $\vartheta = \arg Z$ of BPS states:

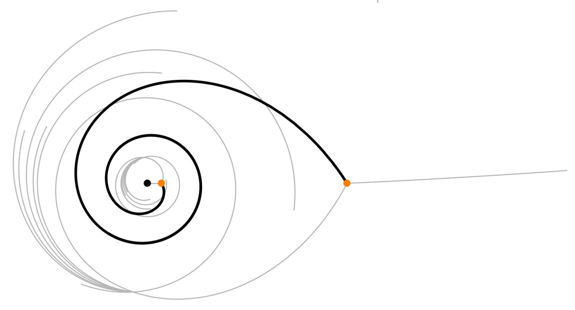
$\Omega(k\mathrm{D0}) = -2$



$\Omega(\mathrm{D2}) = 1$

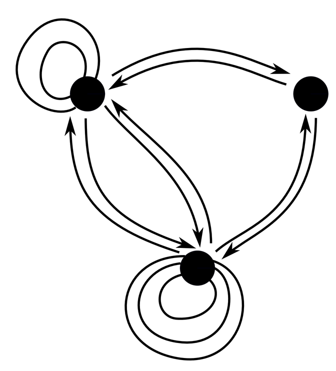


$\Omega(\mathrm{D2\text{-}D0}) = 1$



Knots and Quivers

Example: the trefoil quiver



HOMFLY-PT series from the Quiver series:

$$\sum_{r\geq 0} H_r(a,q)x^r = \sum_{\vec{d}} (-q)^{\vec{d}\cdot C\cdot \vec{d}} \frac{\vec{x}^{\vec{d}}}{(q^2;q^2)_{\vec{d}}}$$

Open topological strings:

- Basic disks for each node
- Boundstates fixed by linking numbers
- Higher genus curves from disks!

3d $\mathcal{N} = 2$ QFT:

- Abelian CS-matter theory $T[Q]$
- Skein relations→duality $T[Q] \simeq T[Q']$

Future Directions

- Use exponential networks to study geometries with compact four-cycles: rich wall-crossing behavior is expected to take place, other techniques become prohibitively challenging.
- Explore the interplay between Exponential Networks and 3d BPS spectra: generalize the relation between Stokes graphs and Exact WKB analysis arising in the reduction on S^1.
- Extend the quiver description of open topological strings beyond knot conormals: e.g. as a way to resum topological vertex results in closed form. Among the advantages: fixing a prescription for the quantization of the mirror geometry.

References

[1] T. Ekholm, P. Kucharski and P. Longhi, "Multi-cover skeins, quivers, and 3d $\mathcal{N} = 2$ dualities," JHEP **02** (2020), 018
[2] S. Banerjee, P. Longhi and M. Romo, "Exponential BPS graphs and D brane counting on toric Calabi-Yau threefolds: Part I," [arXiv:1910.05296 [hep-th]].
[3] S. Banerjee, P. Longhi and M. Romo, "Exploring 5d BPS Spectra with Exponential Networks," Annales Henri Poincare **20** (2019) no.12, 4055-4162
[4] T. Ekholm, P. Kucharski and P. Longhi, "Physics and geometry of knots-quivers correspondence," [arXiv:1811.03110 [hep-th]].
[5] H. Ooguri and C. Vafa, "Knot invariants and topological strings," Nucl. Phys. B **577** (2000), 419-438
[6] P. Kucharski, M. Reineke, M. Stosic and P. Sułkowski, "BPS states, knots and quivers," Phys. Rev. D **96** (2017) no.12, 121902
[7] R. Eager, S. A. Selmani and J. Walcher, "Exponential Networks and Representations of Quivers," JHEP **08** (2017), 063
[8] A. Klemm, W. Lerche, P. Mayr, C. Vafa and N. P. Warner, "Selfdual strings and N=2 supersymmetric field theory," Nucl. Phys. B **477** (1996), 746-766
[9] D. Gaiotto, G. W. Moore and A. Neitzke, "Spectral networks," Annales Henri Poincare **14** (2013), 1643-1731