

**Title:** Introduction to Statistical Mechanics (Yvan Velenik)

**Abstract:** The aim of this course is to introduce the students to the mathematical analysis of classical lattice spin systems. Several fundamental questions will be addressed and illustrated in the simplest relevant situations. Among others, the following topics should be covered:

- \* The phase diagram of the Ising model: correlation inequalities, infinite-volume Gibbs states, uniqueness and non-uniqueness, etc.
- \* The discrete Gaussian Free Field: random walk representation, existence/nonexistence of infinite-volume Gibbs states, etc.
- \* Two-dimensional models with continuous symmetry: the Mermin-Wagner theorem, decay of correlations, etc.
- \* Reflection positivity (chessboard estimate, infrared bound) with applications to  $O(N)$  models.

The course will be based on Chapters 3, 8, 9 and 10 of the book

*Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction*  
Sacha Friedli and Yvan Velenik  
Cambridge University Press, 2018

which can be downloaded at the following address:

<https://www.unige.ch/math/folks/velenik/smbook>

**Prerequisites:** probability theory; real and complex analysis

**Exam :** oral

**Title:** Introduction to Statistical Mechanics (II): The example of the Ising model (Hugo Duminil-Copin)

**Abstract:** This course is a follow-up of Introduction to Statistical Mechanics (1). the course will describe delicate properties of one of the most fundamental models of statistical physics, namely the Ising model. Doing so, we hope to illustrate other important aspects of the theory of phase transition. We will describe the properties of the phase transition on the hypercubic lattice. Among others, we will discuss the following topics:

- \* Graphical representations of the Ising model (Random-current representation, Fortuin-Kasteleyn percolation).
- \* Conformal invariance in 2D.
- \* Mean-field behavior in high dimensions.
- \* Classification of Gibbs states.
- \* Sharpness of the phase transition.
- \* Glauber dynamics.

**Exam :** written

**Title:** Quantum Mechanics for mathematicians (Anton Alekseev)

**Abstract:** In this course, we will introduce the basic notions of quantum theory such as the Hilbert space, observables and time evolution. We will start with examples where the Hilbert space is finite dimensional such as particle spins and spin chains. We will continue with more complicated examples which involve the Sturm-Liouville operator and the Schrödinger equation.

If time permits, we would like to introduce the following interesting topics: Bethe Ansatz for quantum spin chains, Bell inequalities, semiclassical picture and the WKB approximation, continuous spectrum and scattering theory.

**Pre-requisites:** linear algebra, differential calculus, some knowledge of classical mechanics is desirable but the facts which are needed will be covered in the course.

**Title:** An introduction to Topological Field Theory (Marcos Marino)

**Abstract:** this course will provide an introduction to Topological Field Theory. We will start with supersymmetric quantum mechanics and move on to topological field theories in dimensions two, three and four (if time permits). I will mostly use a heuristic physics approach, therefore some familiarity with quantum theory will be useful.

**Exam :** written

**Title :** Random growth and Loewner evolution (Stanislav Smirnov & Amanda Turner)

**Abstract :**

Random growth arises in physical and industrial settings, from cancer to polymer creation. In this course we will look at mathematical models for random growth and the features that they exhibit. We will focus on models in 2-dimensions where techniques from complex analysis, such as Loewner evolution, have enabled recent progress. Topics will include :

1. Lattice based models for random growth, including diffusion-limited aggregation (DLA) for mineral aggregation, the Eden model for biological growth, and dielectric-breakdown models.
2. Complex Brownian motion, conformal mappings and the Loewner differential equation
3. Off-lattice models from random growth, including the Hastings-Levitov models, and Schramm-Loewner evolution (SLE )
4. Scaling limits of random growth models

**Reference :** G.F. Lawler, Conformally invariant processes in the plane, Mathematical Surveys and Monographs no. 114, American Mathematical Society, Providence, RI, 2005

**Pre-requisites :** probability theory, real and complex analysis

**Exam :** oral

**Title:** Random Matrices and Universality I and II (Antti Knowles and Johannes Alt )

**Abstract:** This course introduces the students to the theory of random matrices and familiarizes them with some important recent developments. A tentative list of topics includes the following:

- Wigner matrices and the semicircle law
- The Green function, local laws, eigenvalue rigidity, eigenvector delocalization
- Comparison arguments and universality of eigenvalue statistics
- Dyson Brownian motion and applications to universality
- Distribution of eigenvectors
- Sparse matrices and random graphs
- General Wigner-type matrices and the matrix Dyson equation.

Prerequisites: basic probability, analysis, and linear algebra.

**Exam :** oral

**Title :** Knots and Quantum Groups: Theory and Computations without Representations (Dror Bar Natan)

**Abstract :** Our class will run in two parallel streams: "Theory" and "Practice".

In the "Theory" stream we will start with knot theory and mention a few of the main problems that arise within it. This will lead us to learn about and covet Hopf algebras with certain properties, more or less what is known as "quantum groups". Quantum groups are often studied via their representations, but we will do better! We will find that quantum groups have "solvable approximations" that can be understood in terms of the almost-category of "Gaussian Differential Operators", leading to better relations with topology and enabling more effective computations. The "Practice" stream will happen in a computer lab and in it everything theory will immediately become practice. Along the way we will learn how to implement sophisticated mathematics in Mathematica.

**Prerequisites :** Absolutely no fear of linear algebra: quotients, duality, tensor products, symmetric algebras, etc. No fear of Lie algebras. Having heard of universal enveloping algebras and the PBW theorem. Having seen Gaussian integration.

**Title:** Quantum link invariants (Anna Beliakova) Mini-course

**Abstract:** Quantum invariants are more than just topological invariants needed to tell objects apart. They build bridges between topology, algebra, number theory and quantum physics helping to transfer ideas, and stimulating mutual development.

In this course I will introduce these objects from different perspectives: skein and representation theoretic. We will start with the Jones polynomial, study its properties, and then move to the categorification of this polynomial discovered by Khovanov.

Basic knowledge of algebraic topology is of advantage, however all necessary ingredients will be introduced in the course.

**Title :** Quantum information theory

**Abstract:** Quantum theory describes physical phenomena in an equally fascinating and counter-intuitive manner. The goal of quantum information theory is to use these striking quantum properties for applications in information processing. For instance, quantum cryptography allows two users to exchange secret messages with guaranteed privacy, a task that would be impossible to achieve in classical physics. Moreover, quantum information theory offers a fresh perspective and a deeper understanding of the foundations of quantum physics.

The goal of this course is to present the basic concepts and methods of this field, making links with current research.

**Prerequisites:** basic knowledge of quantum physics.

**Exam :** oral