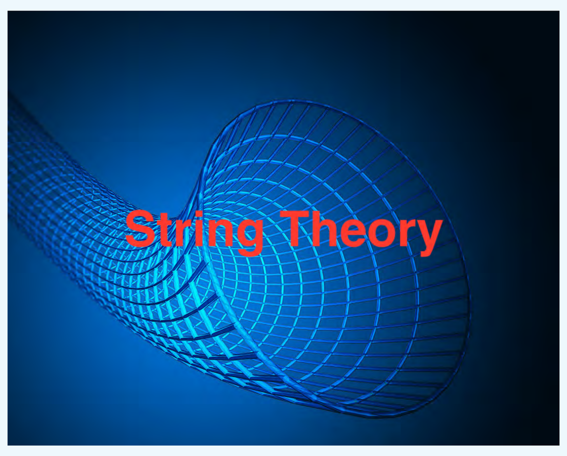


Eigenstate Thermalisation & Ergodicity in Low Dimensional Theories

Pranjal NAYAK
University of Geneva

SwissMAP — The Mathematics of Physics — National Centre of Competence in Research

FNSNF — Swiss National Science Foundation

1. Background & Motivation

- The Eigenstate Thermalization Hypothesis (ETH) provides a possible way to explain thermalization in a quantum system.
- It states that expectation value of a 'typical' operator in an Eigenstate is approximately same as its expectation value in a micro-canonical ensemble, up to exponentially suppressed corrections in system size/entropy.
- The SYK model, a model of large N number of Majorana Fermions, provides a playground to test ETH analytically. It has a universal low energy limit described by 'Schwarzian theory'.
- We analytically demonstrate ETH in the universal Schwarzian theory, as well as in the SYK in various limits.

- Random Matrix Theory (RMT) is a canonical example of a system obeying ETH.
- Wigner-Dyson (WD) spectral statistics is a notable attribute of RMT. These spectral statistics are also underlying property of Quantum Chaotic systems.
- Away from the ergodic regime, where WD statistics hold, the corrections are non-universal.
- We study the behaviour of the operators in the ergodic regime of the SYK model.

- Using the AdS/CFT correspondence we can relate the black hole formation in a theory of gravity to thermalisation in the dual field theory.
- Our ultimate goal is to develop a microscopic understanding of black hole formation by improving our understanding of thermalisation.

2. Introduction to the SYK model

1. SYK Hamiltonian [1]:

$$H = (i)^{\frac{q}{2}} \sum_{1\le i_1<i_2\dots<i_q\le N} j_{i_1\cdots i_q}\psi_{i_1}\psi_{i_2}\dots\psi_{i_q}, \quad \langle j^2_{i_1\cdots i_q}\rangle = \frac{J^2(q-1)!}{N^{q-1}}. \tag{1}$$

2. IR Effective action: In the IR limit, $J|\tau| \to \infty$, reparametrization is an emergent symmetry of the action. This is spontaneously broken by the ground state, and explicitly broken by introducing large but finite J. The low-energy effective action is given by Schwarzian action:

$$S_{Sch} \sim \frac{N}{J}\int d\tau \,\{f(\tau),\tau\}, \quad \{f(\tau),\tau\} = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2 \tag{2}$$

3. Computation of the correlation functions: In the large N limit, correlations functions of fermions receive contributions from,
   - Sum over ladders
   - Sum over $f(\tau)$ modes

3. ETH in the Conformal Limit

1. In the IR limit, the conformal four-point function can be computed exactly by summing over the ladder diagrams.
2. Spectrum of operators in this limit is:

$$\mathcal{O}_n = \frac{1}{\sqrt{N}}\sum_{k=0}^{2n+1}\left[\sum_{i=1}^{N} d_{nk}\,\partial^k\psi_i\,\partial^{2n+1-k}\psi_i\right], \tag{3}$$

with conformal weights, $h_n = 2n+1+2\epsilon_n$

3. Correlation functions of operators $\mathcal{O}_n$ were computed in [2] for arbitrary values of m, n, k and q.
4. We study the three-point function, $\langle\mathcal{O}_m\mathcal{O}_k\mathcal{O}_n\rangle$ in the limit $m, n \gg k \sim \mathcal{O}(1)$, (where $E = \frac{m+n}{2}$, $d = n-m$), [5],

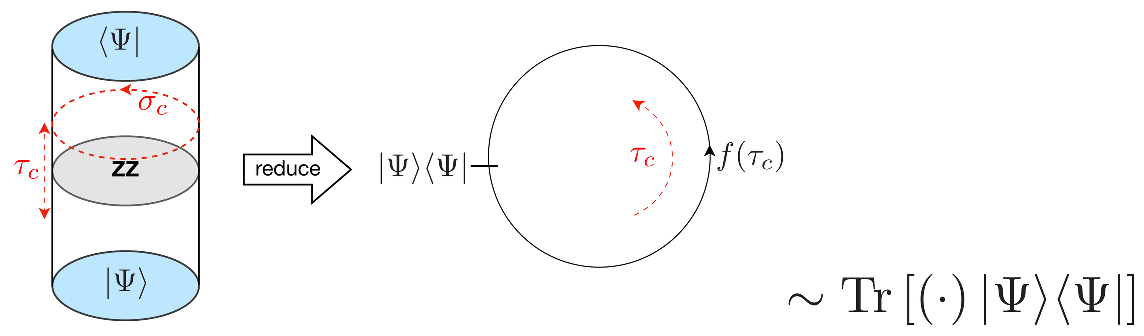
$$\langle\mathcal{O}_m\mathcal{O}_k\mathcal{O}_n\rangle = \mathfrak{g}(E,d)\times\left[2^{-2(2E-1)}\sqrt{2\pi E}\frac{\Gamma(4E-1)}{\Gamma(2E-d)\Gamma(2E+d)}\right] \tag{4}$$

$$\xrightarrow{E\to\infty} \boxed{f_k(E)\,\delta_{m,n} + e^{-E\ln 2}\,R(E,d)}$$

This is the same behaviour for ETH as discussed in [3]

4. ETH in the Schwarzian Limit

1. The Schwarzian action, (2), can be obtained from a dimensional reduction from 2-dimensional boundary Liouville theory, [4, 6].

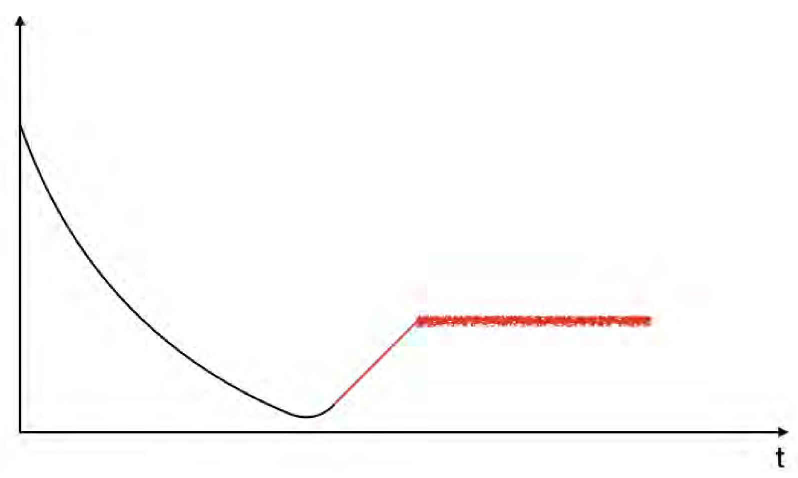


2. Depending on boundary conditions of the boundary Liouville CFT, we obtain an operator/density matrix insertion in the Schwarzian path integral. These correspond to pure coherent states of the Schwarzian theory, [6].
3. Summary of thermal nature of different states:

state	2D picture	class	ETH	λ
$\lvert E(k)\rangle$	ZZ	parabolic	✔	$2\pi T_{\text{ETH}}$
$\lvert E_{r-}\rangle$	FZZT	elliptic	✔	$2\pi T_{\text{ETH}}$
$\lvert E_{r+}\rangle$	FZZT	hyperbolic	✘	$\in i\mathbb{R}$
$\mathcal{O}_{\ell_H}\lvert 0\rangle$	ZZ	parabolic	✔	—

5. Ergodicity in the SYK model

1. Using the 1st quantized Hamiltonian of the SYK model, the ergodic limit of the SYK model can be found, [7]. In this limit, one can analytically extract WD from the SYK Hamiltonian.
2. Operator correlation functions have a characterstic behaviour in this regime,



3. In an ongoing work, we study the ergodic limit of the operator correlation functions in the SYK model.

References

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