Non-boost invariant hydrodynamics

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Background

A system that is in a state of local thermodynamic equilibrium evolving to its global equilibrium state is described by hydrodynamics. Equations of hydrodynamics are formulated in a derivative expansion. The possible terms that may appear in this expansion are **restricted by the symmetries**.

We formulate the complete first-order theory of hydrodynamics invariant under **time translations**, the **Euclidean group of spatial symmetries** and containing a **conserved charge or particle number**, i.e. total symmetry group $\mathbb{R}_t \times \mathrm{ISO}(d) \times \mathrm{U}(1)$ [1]. The thermodynamic functions and transport coefficients that we find are all functions of

- temperature $T(t, x^i)$
- chemical potential $\mu(t, x^i)$
- square of the velocity field $v^2(t, x^i)$

We hope to apply this framework in:

- distinguishing quasi-normal modes from spatial collective modes [2]
- describing the electron fluid of graphene at finite carrier density
- biophysics of self-propelled organisms, e.g. bird flocking

Entropy current

One of the physical requirements of any theory of hydrodynamics is **positivity of entropy production**. This can lead to constraints on transport coefficients.

We construct the most general expression for the entropy current S^μ consistent with the symmetries at hand, up to first derivative order, and then ensure that $\partial_\mu S^\mu \geq 0$.

There are also linearly independent combinations of transport coefficients that do not enter $\partial_\mu S^\mu$. They are responsible for effects which are nonuniform and **non-dissipative**. We find 9 such combinations.

Example: shear modes and shear viscosity: consider shear-type velocity perturbation around uniform flow,

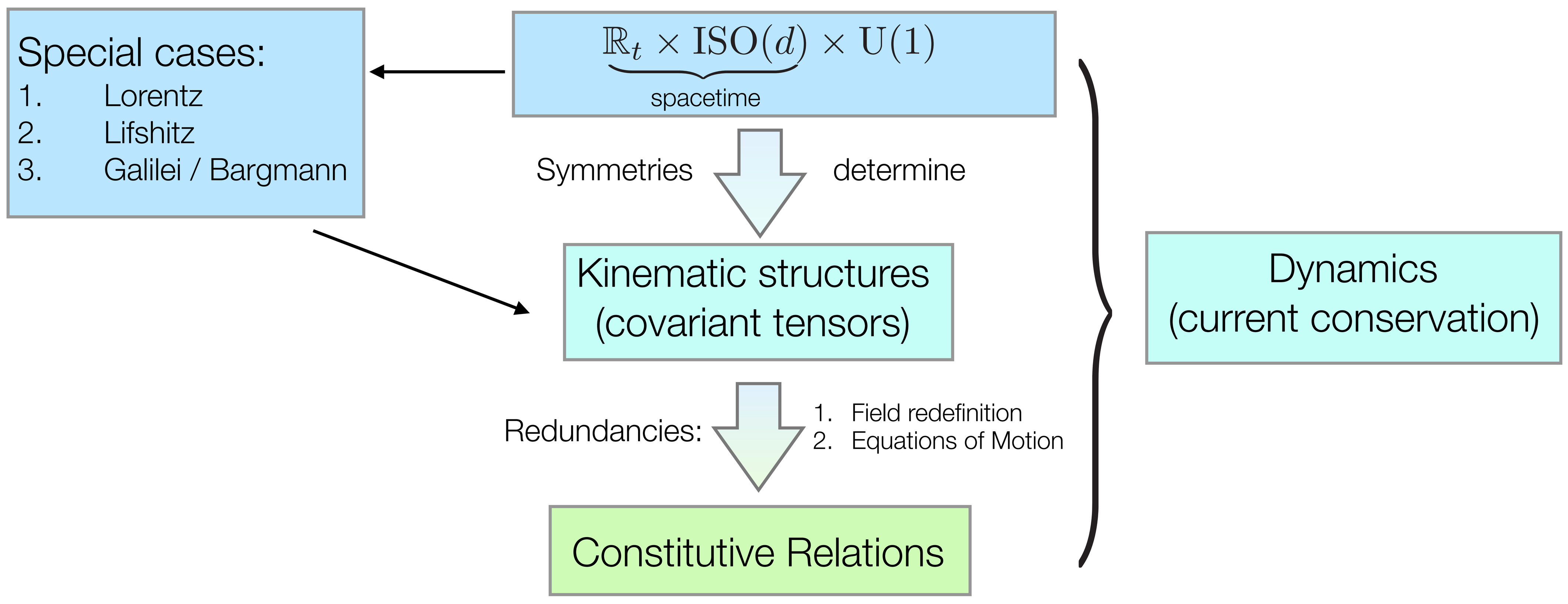
$$T(t,\vec{x}) = \bar{T}, \quad \mu(t,\vec{x}) = \bar{\mu}, \quad v^i = \bar{v}^i + \delta v^i(t, k\cdot\vec{x}),$$

where k^i is a spatial wavevector and $k \cdot \delta v = \bar{v} \cdot \delta v = 0$. The constraints coming from the entropy current analysis are

$$\bar{\eta} \geq 0, \qquad \bar{\eta} + v^2(\bar{\pi} + \gamma_6 - \gamma_7 + \gamma_{13}) \geq 0.$$

These also coincide with those arising out of the requirement of dynamical stability.

Method



We write down all possible kinematic structures compatible with the symmetry $\mathbb{R}_t \times \mathrm{ISO}(d) \times \mathrm{U}(1)$ at first order:

scalars	$[v^k\partial_k T],\ v^k\partial_k v^2,\ v^k\partial_k \frac{\mu}{T},\ [\partial_t T],\ \partial_t v^2,\ [\partial_t \frac{\mu}{T}],\ \partial_k v^k$
vectors	$[\partial_i T],\ \partial_i v^2,\ \partial_i \frac{\mu}{T},\ \partial_t v^i,\ v^k\partial_k v^i,\ v^i\cdot$ (scalars)
tensors	$\sigma_{ij},\ v^{(i}\cdot(\text{vectors})^{j)},\ \delta^i_j\cdot$ (scalars)

Constitutive relations

Ideal fluid: the constitutive relations for an ideal fluid in this symmetry class were written first by [3, 4]. Examples of these relations:

$$T^{(0)0}{}_0 = -\mathcal{E}, \quad T^{(0)0}{}_j = \rho v^j,$$
$$T^{(0)i}{}_0 = -(\mathcal{E}+P)v^i, \quad T^{(0)i}{}_j = P\delta^i_j + \rho v^i v^j$$

First order: we impose conditions equivalent to the so-called **Landau frame** conditions. We find in total 29 transport coefficients [1]:

$$\bar{\eta}, \bar{\zeta}, \bar{\sigma}, \bar{\alpha}, \bar{\gamma}, \bar{\pi}, \quad \gamma_1, \ldots, \gamma_{23}$$

Examples of constitutive relations at first order:

- Energy density

$$\Pi^0{}_0 = \gamma_2 v^k\partial_k v^2 + \left(\gamma_1 v^2 + \frac{\bar{\pi}}{2}\right)\partial_t v^2 + \gamma_3 v^2 \partial_k v^k + (\gamma_4 v^2 - T(\bar{\alpha}+\bar{\gamma}))v^k\partial_k \frac{\mu}{T}$$

- U(1) current

$$\Pi^i = \gamma_{22}\partial_i v^2 + \gamma_{23} v^k \partial_k v^i - T\bar{\sigma}\partial_i \frac{\mu}{T} + (\bar{\alpha}-\bar{\gamma})\partial_t v^i$$

Special cases

Imposing additional symmetries further constrains our transport coefficients. Examples:

Lorentz boosts: the transport coefficients are completely determined by only 4 free functions of two variables: the shear viscosity η, bulk viscosity ζ, conductivity σ and χ. Each of these are arbitrary functions of $\tilde{T} = \gamma T, \tilde{\mu} = \gamma\mu$, where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. Further reduced to 3 due to positivity of entropy production ($\chi = 0$).

Galilean boosts: starting with a relativistic theory and sending $c \to \infty$, the resulting theory is invariant under *massless* Galilean boosts. We find 3 transport coefficients remaining: η, ζ, σ, each a function of T, μ.

Lifshitz scale invariance: invariance under the inhomogeneous scale transformation

$$t \to \lambda^z t, \quad x^i \to \lambda x^i,$$

for some arbitrary dynamical critical exponent z. A transport coefficient $\gamma_I(T, v^2, \mu)$ with scaling weight w_I must be an arbitrary function of

$$\gamma_I(T, v^2, \mu) = T^{\frac{w_I}{z}}\hat{\gamma}_I\left(\frac{v^2}{T^{\frac{2(z-1)}{z}}}, \frac{\mu}{T}\right).$$

Outlook

- It would be interesting to extend our analysis to include **parity non-invariant effects**.
- We would also like to understand better the **physical interpretation of new transport coefficients** that appear in the constitutive relations.
- Another point of interest would be to see **practical applications** of this framework (e.g. graphene, biophysics)

References

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[3] J. de Boer, J. Hartong, N. A. Obers, W. Sybesma and S. Vandoren,"Perfect Fluids", SciPost Phys. **5** (2018) no.1, 003.

[4] J. de Boer, J. Hartong, N. A. Obers, W. Sybesma and S. Vandoren,"Hydrodynamic Modes of Homogeneous and Isotropic Fluids", SciPost Phys. **5** (2018) no.2, 014.